Goldstein 9.17

Consider $[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0$?

$= A[B,C] - [B,C]A + B[C,A] - [C,A]B + C[A,B] - [A,B]C$

Organizing terms yields

$A[B,C] - [A,B]C + B[C,A] - [B,C]A + C[A,B] - [C,A]B$

$= -ACB + BAC - BAC + CBA - CBA + ACB$

$= \boxed{0}$

The second condition is easier to show even

$[A,B]C + B[A,C]$

$= ABC - \cancel{BAC} + \cancel{BAC} - BCA$

$= \boxed{[A,BC]}$

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2.9.2024